

Void or Dark Energy?

John Moffat

Perimeter Institute

Waterloo, Ontario, Canada

Seminar given at Perimeter Institute

November 17, 2009

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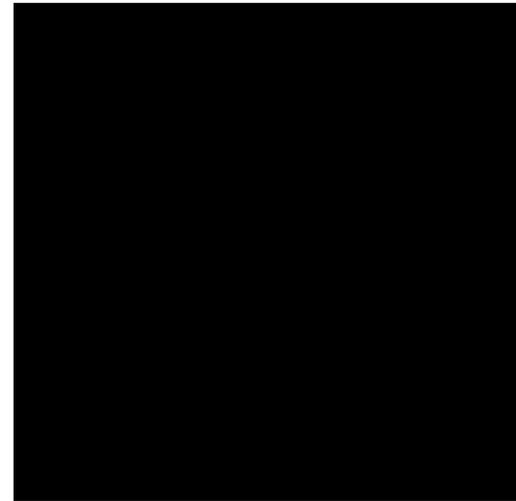
1. Introduction

- The standard CDM model is based on Einstein's gravitational theory (GR) and the cosmological principle: there is no special place in the universe the universe is isotropic and homogeneous.
- The assumptions of an isotropic and homogeneous universe were used to develop the Lemaître-Friedmann-Robertson-Walker (LFRW) expanding universe model, which forms the basis of the standard modern Λ CDM cosmology.

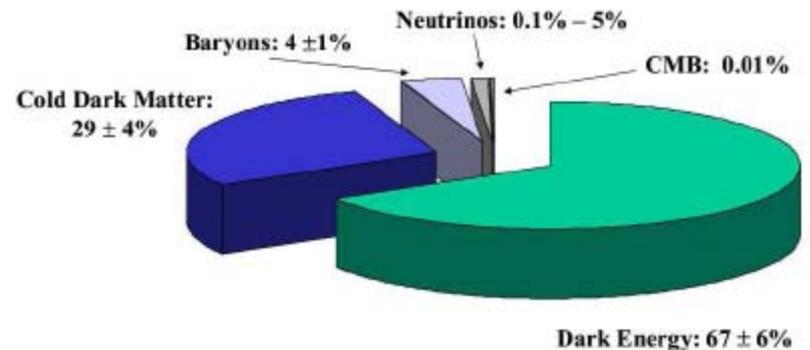
Standard Λ CDM Cosmology

Ingredients of the standard cosmology:

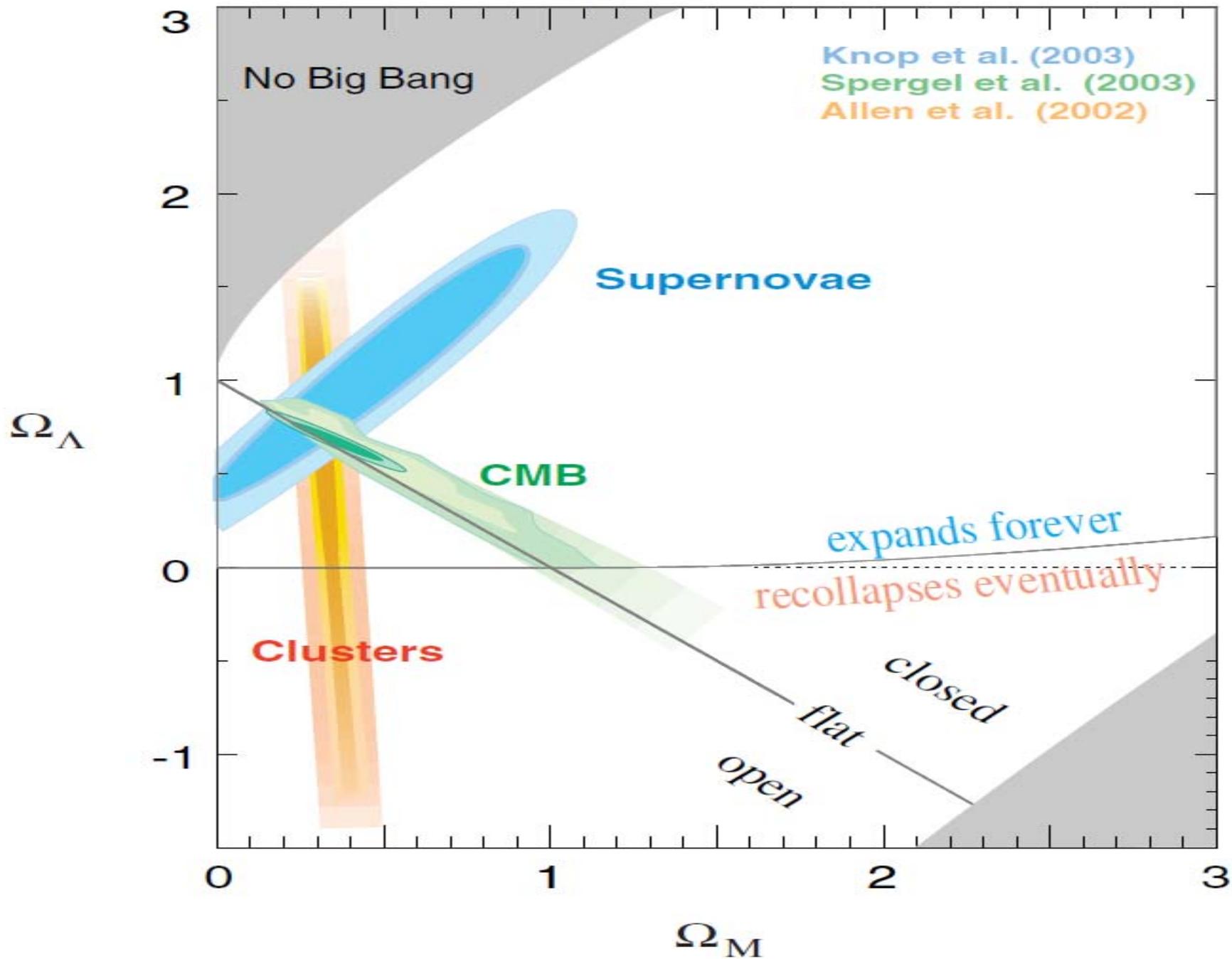
- General Relativity
- Large-scale homogeneity and isotropy
- 5% ordinary matter (baryons and electrons)
- 25% dark matter
- 70% dark energy
- Uniform CMB radiation, $T \sim 2.73$ degrees
- Scale-free adiabatic fluctuations $\Delta T/T \sim 10^{-5}$



Matter and Energy in the Universe: A Strange Recipe



Supernova Cosmology Project

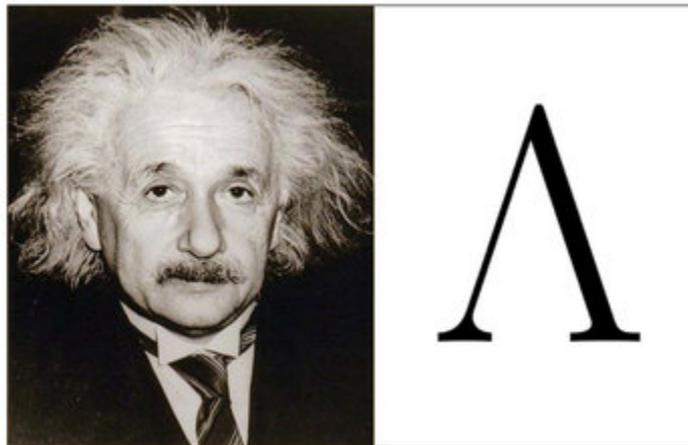


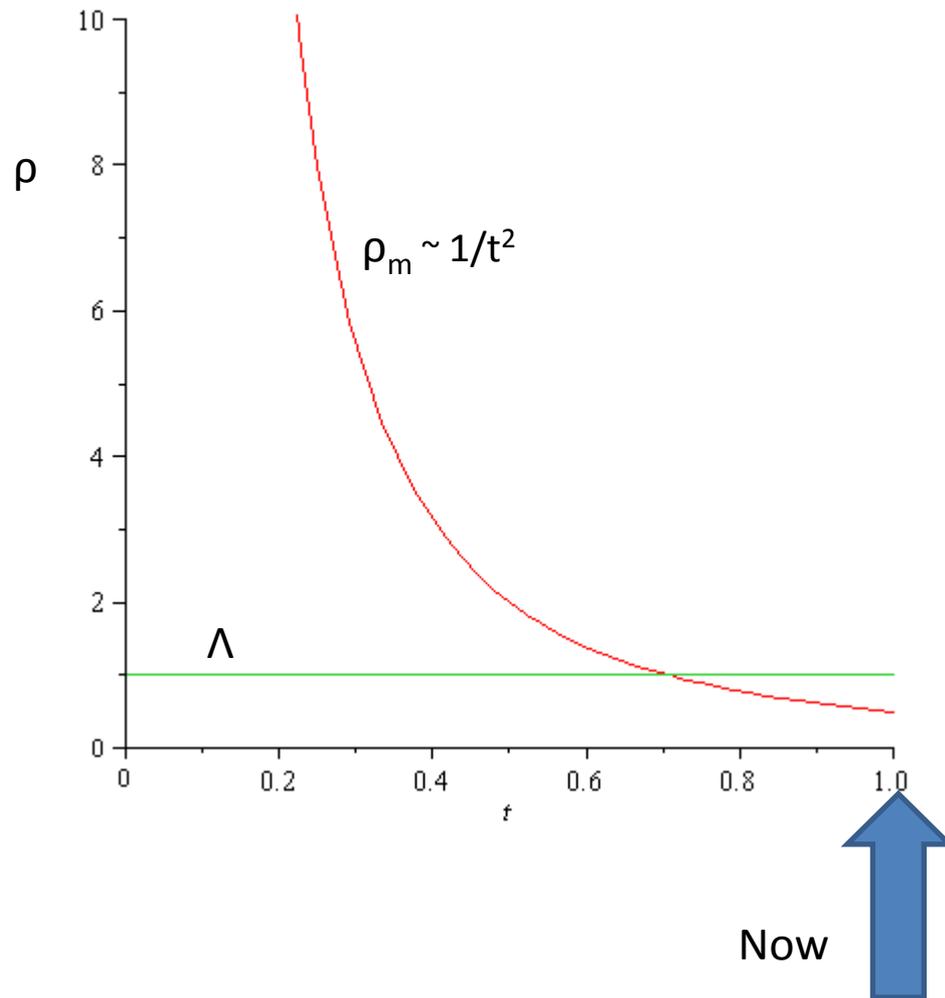
- The standard modern CDM cosmology. Although this model has successfully described the available cosmological observations (in particular, the WMAP CMB data of maximal symmetry, the LFRW geometry of spacetime and that GR is the correct theory of gravity to describe the large scale structure of the universe), the following points apply:
 - A disturbing feature of the standard CDM model is that roughly 96 percent of the universe is invisible. Approximately 30 percent is composed of “dark matter”, while 70 percent is the dark energy that pervades the universe like a modern-day ether, and is responsible for the asserted acceleration of the expansion of the universe invoked to explain the supernovae observations.
 - After several years of searching for the ubiquitous dark matter, no successful detection of dark matter particles has been achieved. By its nature the dark matter particles interact only with gravity, so the existence of dark matter is inferred through gravitational observations such as galaxy rotation curves.

- The dark energy is a mysterious and undetectable uniform substance that has been identified with negative pressure vacuum energy. This interpretation falls prey to the serious lack of understanding of vacuum density in quantum field theory, leading to preposterous degrees of fine-tuning to agree with the “observed” vacuum density associated with the cosmological constant
- To avoid this problem, many modified gravity theories have been proposed that devote themselves to explaining the accelerated expansion of the universe. Many of these theories suffer from maladies that render them unphysical. They can possess ghosts and instabilities and not be able to explain the precise relativistic corrections engendered by GR in the solar system, such as the Cassini spacecraft observation of the Eddington-Robertson PPN parameter $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$.

- An early extensive study of cosmological voids [6, 7] in the exact Lemaître-Tolman-Bondi (LTB) inhomogeneous solution of GR (G. Lemaître, Mon. Not. Roy. Astron. Soc. 91, 490 (1931); Ann. Soc. Sci. Bruxelles, A53, 51 (1933); R. C. Tolman, Proc. Nat. Acad. Sci. 20, 169 (1934); H. Bondi, Mon. Not. Roy. Astron. Soc. 107, 410 (1947) predicted that the luminosity distance in a void solution would depend on the redshift z in a way that deviates from the FLRW model prediction [JWM and D. C. Tatarski, Phys. Rev. D45, 3512 (1992); JWM and D. C. Tatarski, Ap. J. 453, 17 (1995), arXiv:astro-ph/940703.]
- The distance modulus calculated from the void model in 1994-95 suggested an apparent acceleration of the expansion of the universe as inferred by an observer in an FLRW spacetime. **Indeed, four years before the celebrated supernovae SNe Ia observations, it was clear that if the inhomogeneous void scenario is correct, then the observed dimming of the supernovae light was to be expected.**

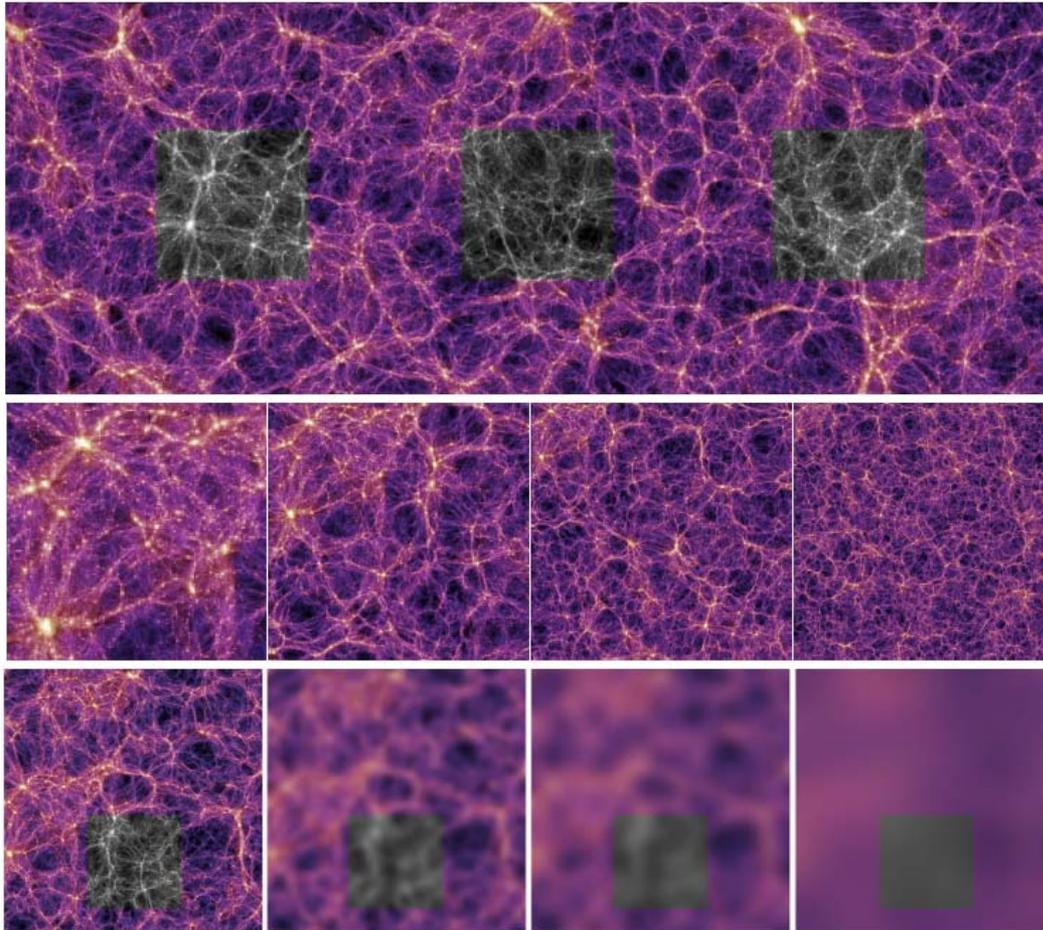
- The void solution demanded that the observer be situated close to the center of the void to maintain the observed isotropy of the large scale structure of the universe. This would imply a violation of the cosmological Copernican Principle [R. R. Caldwell and A. Stebbins, Phys. Rev. Lett. 100 191302, 2008; C. Clarkson, B. A. Bassett and T. Hui-Ching Lu, Phys. Rev. Lett. 101:011301,2008, arXiv:0712.3457].
- The adherence to a dark energy such as the vacuum energy in the CDM model, also suffers from an anti-Copernican principle fine-tuning in coordinate time. The negative pressure dark energy and the associated acceleration of the expansion of the universe began dominating the evolution of the standard model when life first appeared on our planet. This is referred to as the “coincidence” problem.
- Thus, we are faced with having to make a choice between two theoretical evils, the dark energy scenario or the more conservative inhomogeneous void scenario.





- It is difficult with the presently available observational data to distinguish between the idealistic Copernican Principle cosmology and those cosmological models that violate it.
- One ongoing vexing problem with the postulate of a cosmological constant in the CDM standard model is our lack of understanding of the vacuum density in physics. The notorious cosmological constant problem remains unresolved, leading to a huge fine-tuning problem.
- In the standard electroweak model with a Higgs particle the predicted vacuum density is of order 10^{56} times bigger than the vacuum density required in the CDM model. The problem becomes even more pronounced when energy scales reach the Planck energy $\sim 10^{19}$ GeV, resulting in a fine-tuning of order 10^{122} compared to the “observed” cosmological constant Λ .

- A relativistic modified gravity (MOG) theory known as the Scalar-Tensor-Vector Gravity (STVG) theory has been applied successfully to fit astrophysical as well as cosmological data without nonbaryonic dark matter [JWM, JCAP 0603 (2006) 004, arXiv:gr-qc/0506021; JWM and V. T. Toth, Talk given by JWM at the "The Invisible Universe" conference, Paris, France, June 29-July 3, 2009, arXiv:0908.0781; JWM and V. T. Toth, Class. Quantum Grav. 26 085002 (2009), arXiv:0712.1796].
- In contrast to this need to modify gravity to avoid postulating an undetected dark matter, **the void solution to the SNe Ia supernovae observations does not require a modification of Einstein's gravity theory due to its claim that there is no "dark energy"**.
- **An underdense void expands faster than its more dense surrounding galaxies, whereby younger supernovae inside the void would be observed to be receding more rapidly than expected, as compared to older supernovae outside the void.**



Fiddling with the Millennium simulation . Describing the universe as ‘smooth’ doesn’t look right on scales of order $100h^{-1}$ Mpc, shown here in the black and white boxes (top panel). In the central row, we zoom out from $100h^{-1}$ Mpc by a factor of 2 each time (zooming out from the top left corner); only when we get to the last two boxes does it start to look homogeneous-ish, which is $800h^{-1}$ Mpc. (These boxes have the same depth, $15h^{-1}$ Mpc, and it’s the volume that really counts, however.) The averaging problem is shown in the bottom row: how do we go from left to right? Does this process give us corrections to the ‘background’, or is it the ‘background’ itself? C. Clarkson, arXiv: 0911.2601.

- To fit the CMB data and other pertinent cosmological data, the radial size of the void required to fit the supernova data is hundreds of Mpc to Gpc, and we must be close to the center of the void to avoid an unacceptably large CMB dipole.
- Another problem to be considered in the inhomogeneous void model is the assumed initial conditions of the universe. Strictly speaking, simple models of inflation would be inconsistent with the existence of a large inhomogeneous void.
- There exists an alternative solution to the initial value horizon and flatness problems. This is based on a bimetric gravity variable speed of light (VSL) model [JWM, M. A. Clayton and J. W. Moffat, Phys. Lett. B506, 177 (2001), arXiv:gr-qc/0101126]. It has been demonstrated by Magueijo [J. Magueijo, Phys.Rev. D79:043525,2009, arXiv:0807.1689] that this model in conjunction with the Dirac-Born-Infeld model **can predict almost scale invariant primordial fluctuations in agreement with observations without inflation.** This model would not necessarily negate the possibility of large, nonlinear and inhomogeneous voids.

2. Inhomogeneous Friedmann Equations

- For the sake of notational clarity, we write the FLRW line element

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right),$$

- The spherically symmetric inhomogeneous line element is given by

$$ds^2 = dt^2 - X^2(r, t) dr^2 - R^2(r, t) d\Omega^2.$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad T^\mu{}_\nu = (\rho + p) u^\mu u_\nu - p \delta^\mu{}_\nu.$$

$$G_{01} = 0 \quad X(r, t) = \frac{R'(r, t)}{f(r)}$$

- Consider now the Lemaître-Tolman–Bondi (LTB) model for a spherically symmetric inhomogeneous universe filled with dust [see e.g. JWM, JCAP 0605 (2006) 001, arXiv:astro-ph/0505326; JWM, JCAP , 1, 29 (2007), arXiv:astro-ph/0702416; Kenji Tomita and Kaiki Taro Inoue, arXiv:0903.1541; JWM, arXiv:0910.2723; C. Clarkson, arXiv:0911.2601]. The line element is

$$ds^2 = dt^2 - R'^2(t, r) f^{-2} dr^2 - R^2(t, r) d\Omega^2$$

$$2R\dot{R}^2 + 2R(1 - f^2) = F(r) \quad f^2 < 1, = 1, > 1 \quad \rho = \frac{F'}{16\pi R' R^2}.$$

$$M'(r) = \frac{dM}{dr} = 4\pi\rho f^{-1} R' R^2.$$

$$R = \frac{1}{4} F (1 - f^2)^{-1} [1 - \cos(v)], \quad f^2 < 1,$$

$$t + \beta = \frac{1}{4} F (1 - f^2)^{-3/2} [v - \sin(v)], \quad f^2 < 1,$$

$$R = \frac{1}{4} F (f^2 - 1)^{-1} [\cosh(v) - 1], \quad f^2 > 1,$$

$$t + \beta = \frac{1}{4} F (f^2 - 1)^{-3/2} [\sinh(v) - v], \quad f^2 > 1,$$

- In the flat (parabolic) case $f^2 = 1$, we have

$$ds^2 = dt^2 - (t + \beta)^{4/3} \left(Y^2 dr^2 + r^2 d\Omega^2 \right) \quad Y = 1 + \frac{2r\beta'}{3(t + \beta)} \quad \rho = \frac{1}{6\pi(t + \beta)^2 Y}.$$

- We have for finite β that for $t \rightarrow \infty$ the model tends to the flat Einstein–de Sitter case.
- The luminosity distance between an observer at the origin of our coordinate system is given by

$$d_L = \left(\frac{\mathcal{L}}{4\pi\mathcal{F}} \right)^{1/2} = R(t_e, r_e) [1 + z(t_e, r_e)]^2$$

$$ds^2 = dt^2 - R'^2(t, r) f^{-2} dr^2 = 0, \quad d\theta = d\phi = 0$$

$$t = T(r) \quad \frac{dT(r)}{dr} = -\frac{R'}{f} [T(r), r] \quad \frac{d\tau(r)}{dr} = -\tau(r) \dot{R}' [T(r), r]$$

$$\dot{R}' [T(r), r] = \left. \frac{\partial^2 R}{\partial t \partial r} \right|_{r, T(r)} = \left. \frac{\partial R'}{\partial t} \right|_{r, T(r)}.$$

3. Local Void

If we restrict ourselves to spatial scales that have been well probed observationally, i.e. up to a few hundred Mpc, the most striking feature of the luminous matter distribution is the existence of large voids surrounded by sheet-like structures containing galaxies. Early surveys give a typical size of the voids of the order 50–60 h^{-1} Mpc [N. Jackson, Living Rev. Rel. 10, 4 (2007), arXiv:0709.3924 [astro-ph]; M. J. Geller and J. P. Huchra, Science, 246, 897 (1989); F. Hoyle and M. S. Vogeley, Astrophys. J. 607, 751 (2004), astro-ph/0312533; SDSS large scale surveys].

- We study a void with the central density equal to that of an LFRW model with the density parameter $\Omega_0 = 0.2$, asymptotically approaching the Einstein-de Sitter model with $\Omega_0 = 1$ [JWM and D. Tatarski, 1992; JWM and D. Tatarski 1995]:

$$\Omega_{\text{void}}(z) = \frac{\Omega_{\text{min}} + (z/a)^2}{\Omega_{\text{max}} + (z/a)^2}.$$

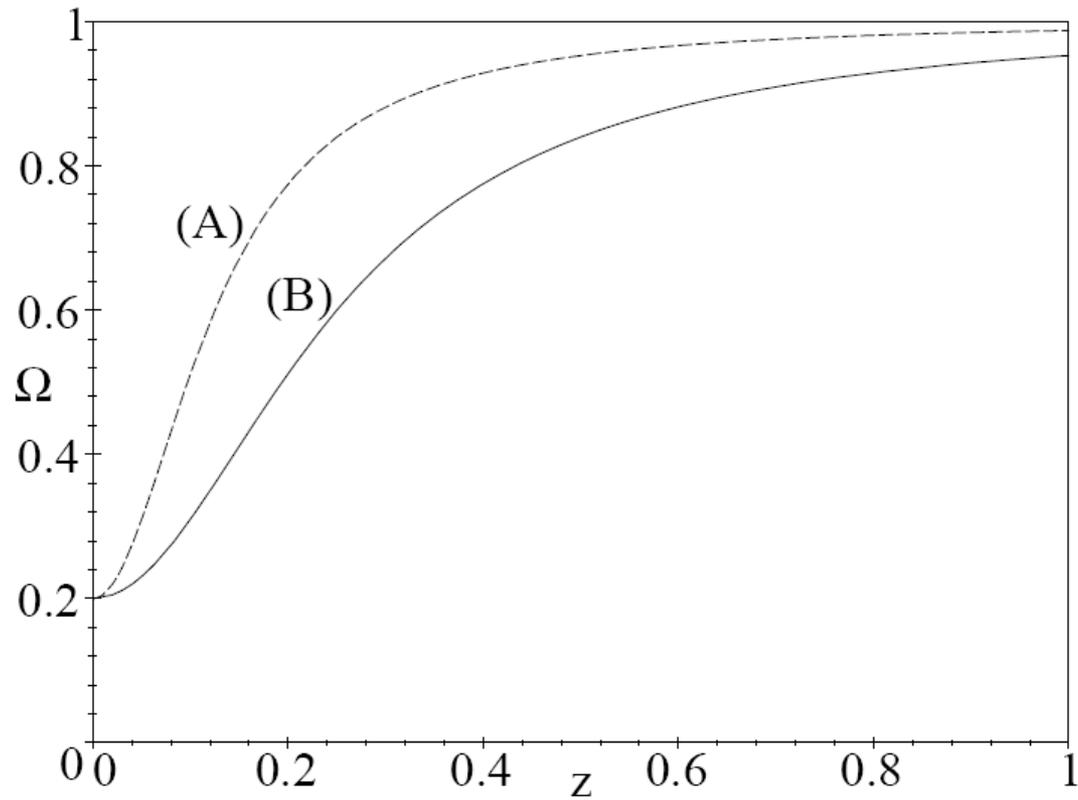
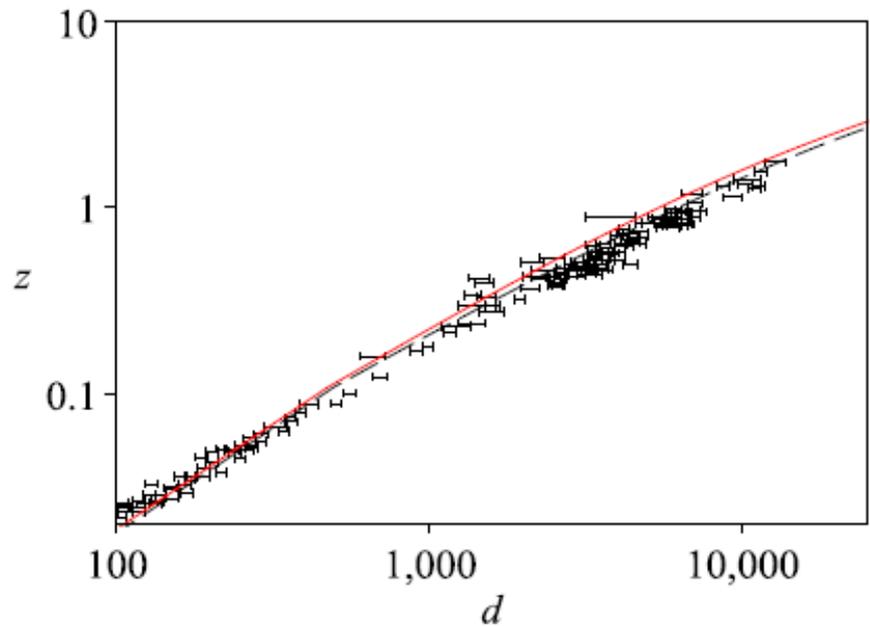
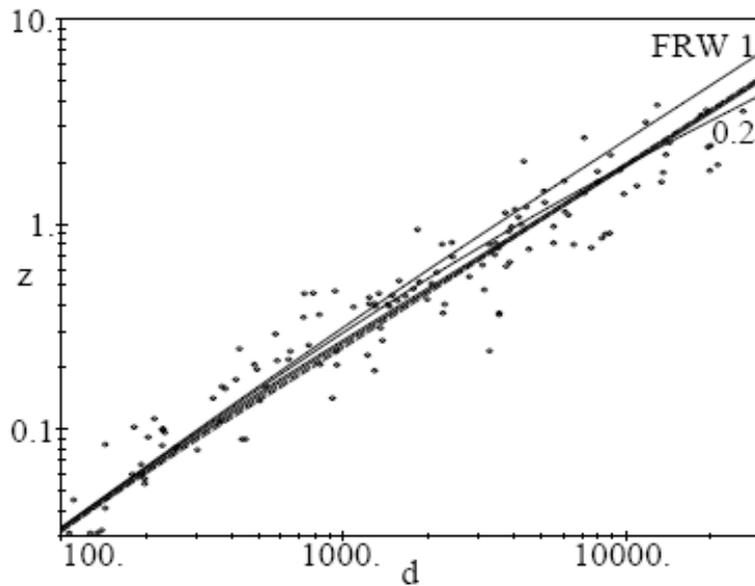
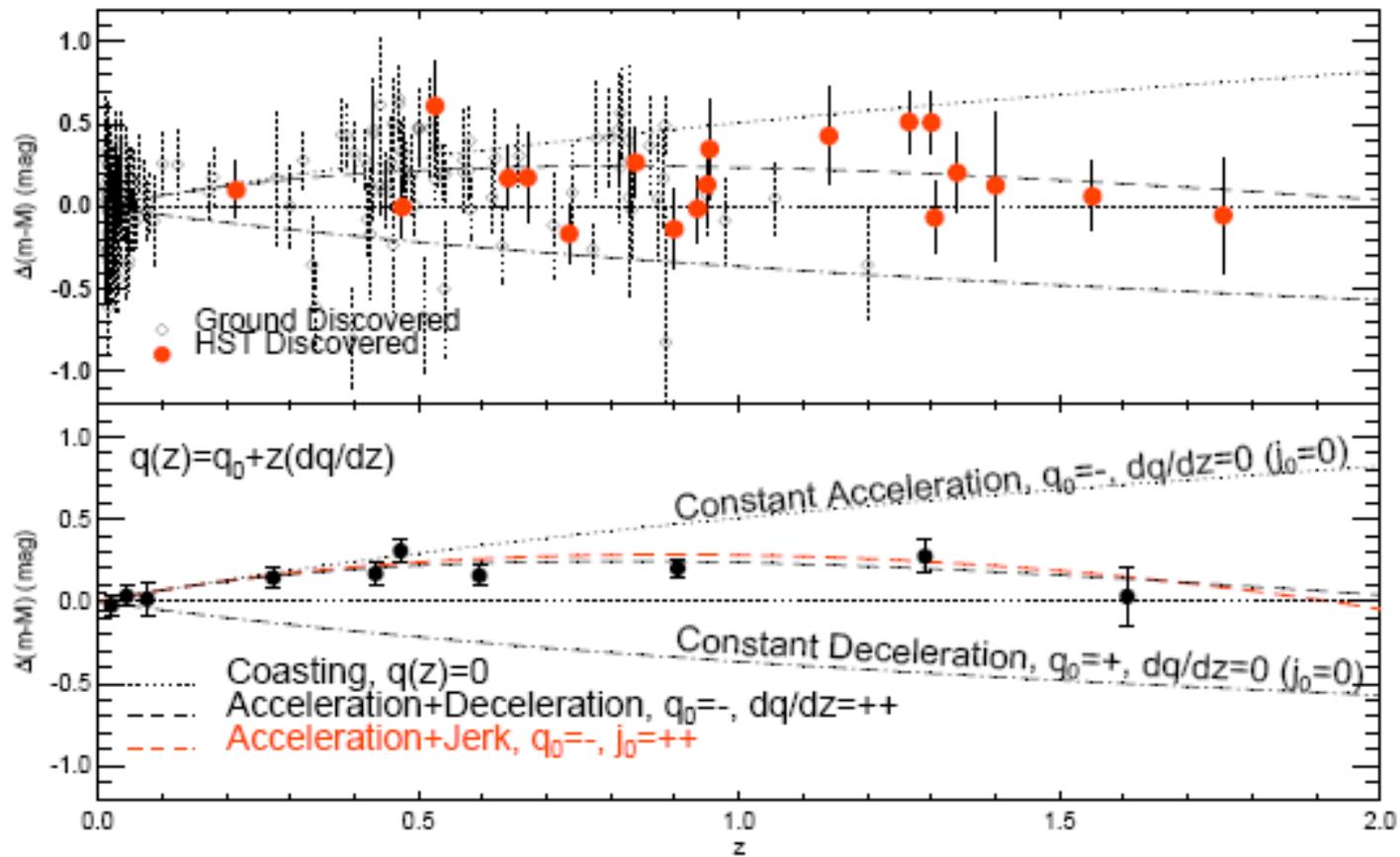


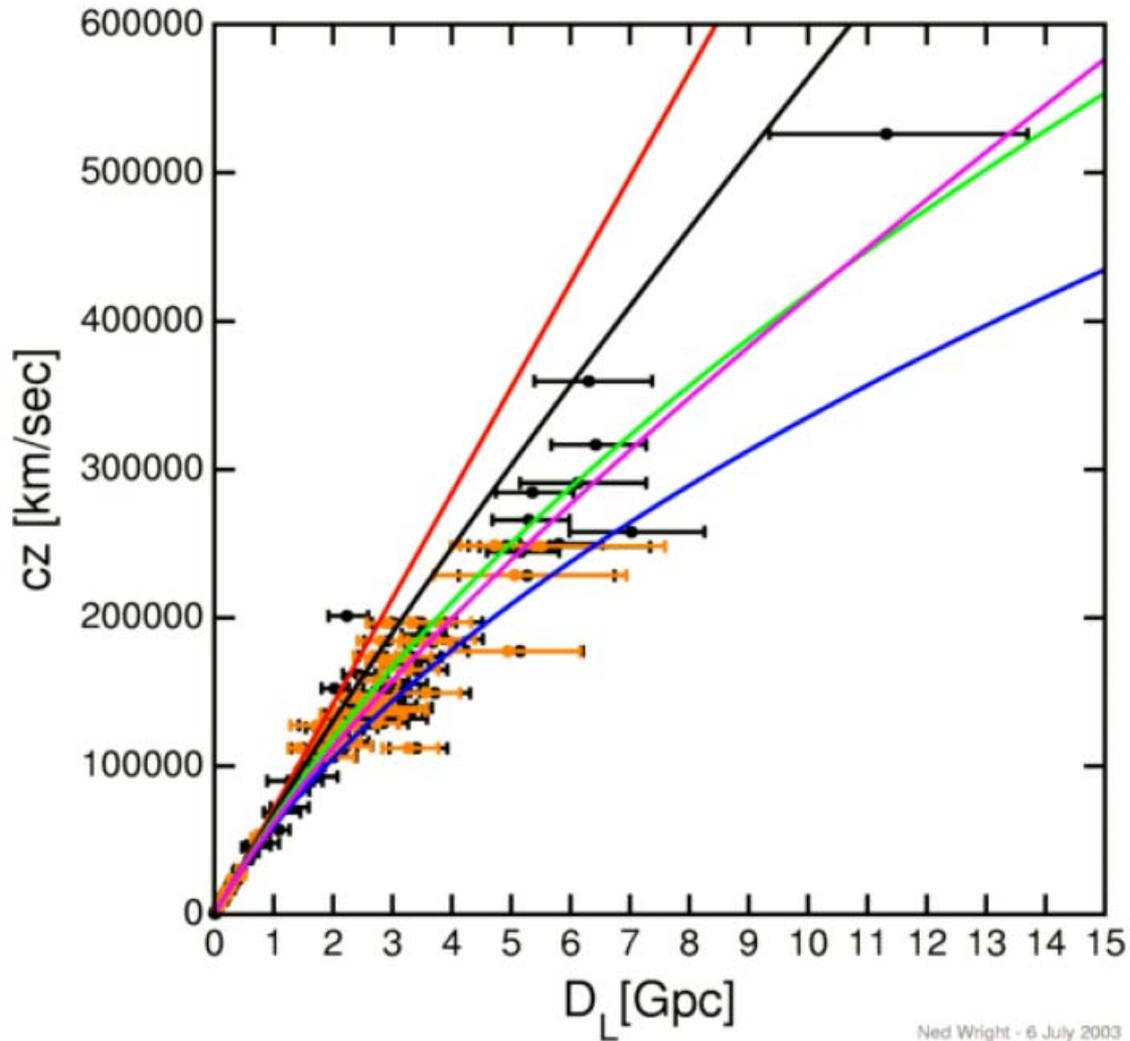
Figure 1: The density distribution void as functions of red shift.



The $\log(z)$ vs. $\log(d_L)$ relations. In the left-hand figure, observational data, denoted by \diamond , are adapted from [S. Lilley, 1993] and the luminosity distance d_L is given in Mpc. In the right-hand figure, the LFRW result for $\Omega_0 = 1$ is shown as a solid red curve, while the CDM result with $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$ is shown as a dashed black curve. SN 1a data are shown for comparison.



Riess et al. 2004



Ned Wright - 6 July 2003

Fig. 2- Recession velocities vs. luminosity distance for SNe Ia. The curves show a closed Universe ($\Omega = 2$) in red, the critical density Universe ($\Omega = 1$) in black, the empty Universe ($\Omega = 0$) in green, the steady state model in blue, and the Concordance model with $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$ in purple. A Hubble parameter $H_0 = 71$ km/sec/Mpc has been used to scale the distances in the plot.

- In the standard LFRW cosmology, we have in the matter-dominated era:

$$\Omega_\Lambda + \Omega_M + \Omega_K = 1 \quad \Omega_\Lambda = \frac{8\pi G\rho_V}{3H_0^2}, \quad \Omega_M = \frac{8\pi G\rho_M}{3H_0^2}, \quad \Omega_K = \frac{K}{a_0^2 H_0^2}$$

- For the flat space standard DCM cosmology we have $\Omega_K = 0$ and the luminosity distance of a source with redshift z is

$$d_L = a_0 r_1 (1+z) = \frac{1+z}{H_0} \int_{\frac{1}{1+z}}^1 \frac{dx}{\sqrt{x^2(\Omega_\Lambda + \Omega_M x^{-3})}}$$

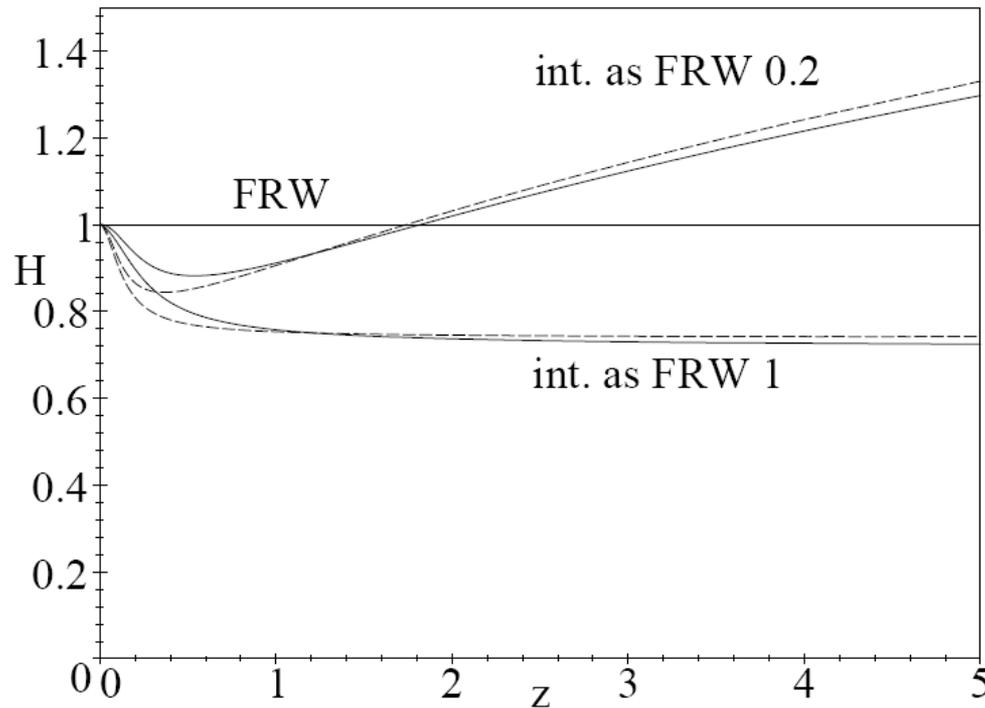
- The distance modulus is the difference between the apparent magnitude m and the absolute magnitude M , given by

$$\mu \equiv m - M = 5 \log_{10} \left(\frac{d_L}{1 \text{ Mpc}} \right) + 25 \quad d_L = (1+z)^2 d_A$$

- We see from the Figure that already four years before the type SNe Ia supernovae measurements were published [JWM and D. Tatarski, 1995], it was clear that if the void model was correct, then the predicted deviations from the LFRW model would lead to unexpected results for red shifts $0.2 < z < 2$, namely, an apparent dimming of the supernovae light, and an apparent acceleration of the expansion of the universe as inferred by an observer using the LFRW model.
- Several recent papers have shown that if the void has a sufficiently large radius, then excellent fits to the supernovae data can be obtained from the LTB void model.
- We have two “Hubble parameters”: $H_r(t, r)$ for the local expansion rate in the radial direction and $H_{\perp}(t, r)$ for expansion in the perpendicular direction:

$$H_r = \frac{\dot{l}_r}{l_r} = \frac{\dot{R}'}{R'}, \quad H_{\perp} = \frac{\dot{l}_{\perp}}{l_{\perp}} = \frac{\dot{R}}{R}$$

- If we lived in a local LTB void and the z versus d_L relation differed from the LFRW one, but we were biased by our theoretical prejudice and interpreted cosmological observations through an LFRW model, we would expect the value of the Hubble parameter to be position and d_L dependent.



The “observed” Hubble constant H in units of the local measurement of H_0 as a function of the red shift z from ref. [JWM and D. Tatarski, 1995].

- Let us recall that in LFRW cosmology the exact result for the Hubble relation d_L versus z in the matter dominated universe is

$$d_L = \frac{1}{H_0 q_0^2} \left[z q_0 + (q_0 - 1) \left(\sqrt{2z q_0 + 1} - 1 \right) \right] \quad q_0 \equiv -\ddot{a}(t_0)/a(t_0) H_0^2$$

- On cosmologically very small distances, we measure the same value of H_0 independently of the model (we call this value “the local measurement”). This stems from the fact that, due to our assumptions, very close to the center ($r \ll 1$) the model is well approximated by the LFRW universe with $\Omega = 0.2$. Obviously, if the universe were locally LTB rather than LFRW, then the Hubble parameter based on the observed LTB values of z and d_L , but inferred through an LFRW relation would be position (redshift) dependent. The dependence of the Hubble parameter H (in units of the H_0 value as measured locally) on the redshift z is shown in the Figure.
- If we interpret the results within the LFRW < 1 framework, the “observed” values of the Hubble constant first decrease with z and then asymptotically increase to some background limit. The position of the minimum in H depends on the size of the LTB void.

4. The Age Problem and Structure Formation

- In the standard CDM model the assumption that the acceleration of the expansion of the universe is caused by a vacuum dark energy, leads to an increase in the age of the universe.
- For the flat LFRW model without a cosmological constant , the present age of the universe at $z = 0$ is

$$t_0 = \frac{2}{3H_0} = 9.3 \left(\frac{70 \text{ km/sec/Mpc}}{H_0} \right) \text{ Gyr.}$$

- When the vacuum energy is included in the calculation of the age of the universe, we get

$$t_0 = (13.4 \pm 1.3) \left(\frac{70 \text{ km/sec/Mpc}}{H_0} \right) \text{ Gyr.}$$

- This is in better agreement with the ages of clusters and stars. In particular, for globular clusters their ages are variously between 11.5 ± 1.3 Gyr and 14.0 ± 1.2 Gyr [Chaboyer, 1998, Carretta, et al 2000, Krauss and Chaboyer 2001]. Schramm [Schramm, 1997] gave as the ages of globular clusters $14 \pm 2(\text{statistical}) \pm 2(\text{systematic})$ Gyr.

- The age estimate in the Λ CDM model is uncomfortably close to the ages of the oldest globular clusters and possibly to the ages of the most distant galaxies observed.
- In our inhomogeneous model, the metric and density are singular on two hypersurfaces:

$$t + \beta = 0, \quad Y = 0, \quad \longrightarrow \quad t_1 = -\beta, \quad t_2 = -\beta - \frac{2r\beta'}{3}.$$

The model is valid only for $t > \Sigma(r) \equiv \text{Max}[t_1(r), t_2(r)]$.

- Here, $t(r) = \Sigma(r)$ defines the big-bang hypersurface in the model. Physically, because the model is pressureless, we interpret $\Sigma(r)$ as the surface on which the universe enters the matter-dominated era.
- In the LFRW model this occurs at the same time t_{eq} when radiation and matter are equal $t_{\text{eq}} \sim 10^4$.
- However, even in a globally flat inhomogeneous model this can occur at different times. We also note that in the limit $t \rightarrow \infty$, the LTB model gives the Einstein-de Sitter universe:

$$ds^2 = dt^2 - t^{4/3}(dr^2 + r^2 d\Omega^2).$$

- At $r = 0$ the big-bang model hypersurface is located at $t(0) = -\beta(0)$. The requirement that $\beta'(r)$ tends to a finite limit as $r \rightarrow \infty$ forces $\beta'(0) = 0$. For an observer at $t(t_0, 0)$ in our void, where t_0 is the time coordinate of constant time hypersurface “now”, the age of the universe is given by

$$t_{LTB} = t_0 + \beta(0) = \frac{2}{3H_{\perp}(t_0, 0)}.$$

- Depending on the choice of $\beta(0)$ we can increase the age of the universe as observed by an earth-based observer at $r = 0$. If we set $\beta(0) = 0$, then

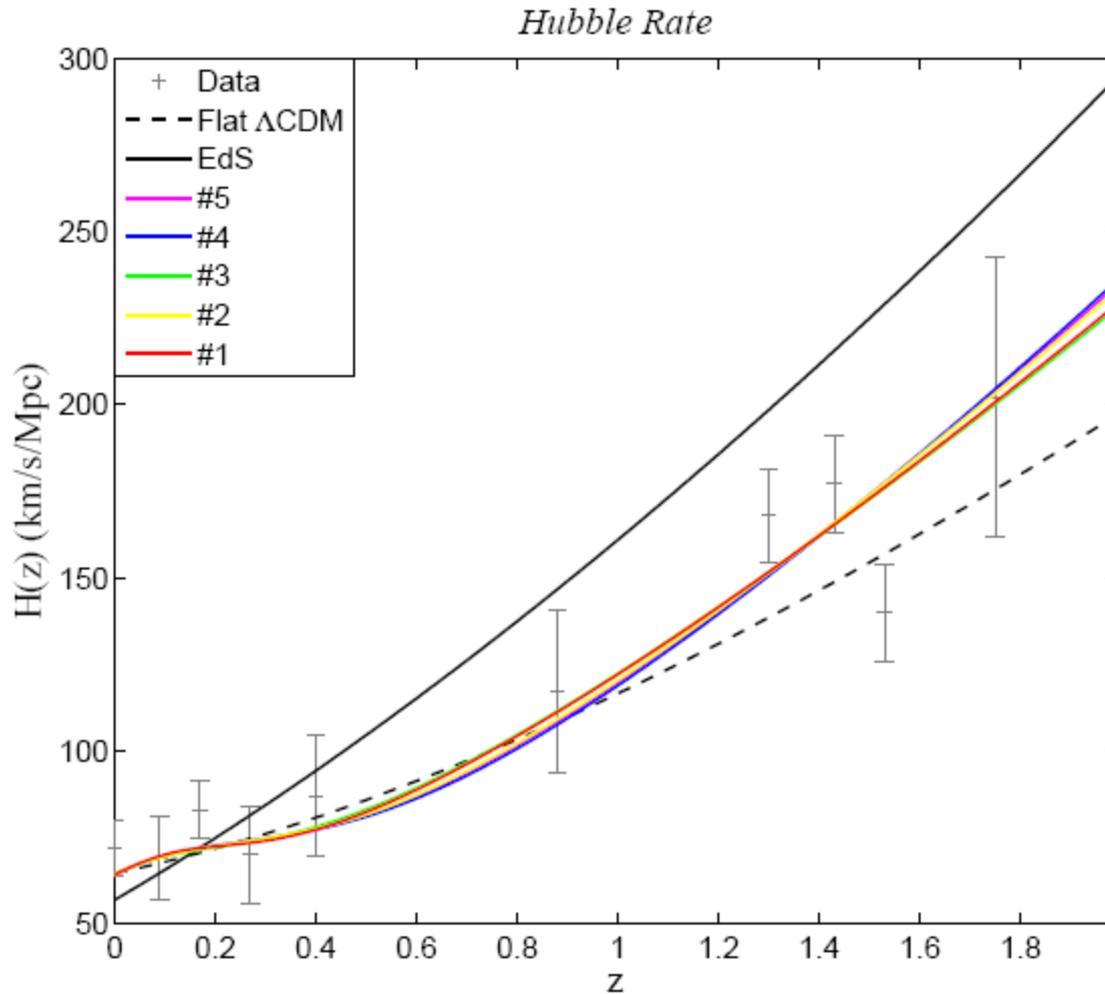
$$t_{LTB} = \frac{2}{3H_{\perp}(t_0)} = \frac{2}{3H_0}.$$

- However, for the outer parts of the universe for large z we can choose $\beta(r)$ and $\Sigma(r)$, so that we are able to obtain an age of the universe much more compatible with the ages of globular clusters and radioactive dating and the ages of the most distant galaxies.

- Let us now turn our attention to structure formation in LTB models. Several authors have investigated the growth of structure in the inhomogeneous LTB model.

- The density contrast in a spatially flat model is described by

$$\delta(\mathbf{x}) = \frac{\delta\rho(\mathbf{x})}{\bar{\rho}} = \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}};$$



Age plot in void models compared with LCDM, E-deSitter and data [S. February et al. arXiv:0909.1479 (2009)].

The autocorrelation function $\xi(\mathbf{r})$ is defined by

$$\xi(\mathbf{r}) = \langle \delta(\mathbf{x} + \mathbf{r})\delta(\mathbf{x}) \rangle.$$

$$\xi(\mathbf{r}) = \frac{1}{(2\pi)^2 V} \int d^3k |\delta_k|^2 \exp(-i\mathbf{k}\cdot\mathbf{x}), \quad |\delta_k|^2 = V \int d^3r \xi(\mathbf{r}) \exp(i\mathbf{k}\cdot\mathbf{r}).$$

- In our inhomogeneous and isotropic case (we as an observer are close to the center of the void) $\xi(\mathbf{r}) = \xi(r)$.
- The density perturbations for the growing mode in the LFRW model obey the equation:

$$\delta_{LFRW} = \delta_{LFRW}(t_{eq}) \left(\frac{t_{LFRW}}{t_{eq}} \right)^{2/3}$$

- t_{LFRW} denotes the time from the initial singularity to a given value of time coordinate t , and t_{LFRW} is the same everywhere in the LFRW model.
- In our inhomogeneous isotropic flat model we have [JWM and D. Tatarski, 1992]:

$$\delta_{LTB}(t, r) = \delta_{LTB}(t_\Sigma, r) \left(\frac{t_{LTB}}{t_\Sigma(r)} \right)^{2/3}$$

where $t_{LTB}(r) = t - (r)$ is the time from the initial singularity.

We also assume that $\delta_{LFRW}(t_{eq}) = \delta_{LTB}(t_{\Sigma}(r), r)$ and $t_{eq} = t_{\Sigma}(r)$ for all r .

- We now find that there is an amplification of the FLRW perturbation growth in our inhomogeneous model

$$\delta_{LTB}(t, r) = \left(\frac{t_{LTB}(r)}{t_{LFRW}} \right)^{2/3} \delta_{LFRW}(t).$$

- The larger $t_{LTB}(r)$ for a given r , the more the structure growth has developed.
- For the correlation function we have

$$\xi_{LTB} = \left(\frac{t_{LTB}}{t_{LFRW}} \right)^{2/3} \xi_{LFRW}(r).$$

The amplification of the structure growth in our model can influence the estimated late-time integrated Sachs-Wolfe (ISW) effect as compared to the standard LFRW model. **Moreover, the size of t_{LTB} can increase the time when structure growth of primordial perturbations enter the horizon.**

5. Can the Void Model Agree with Cosmological Data?

- The void model must fit the CMB WMAP data. Various authors have attempted to fit an LTB model to the WMAP data [see e.g., Kenji Tomita and Kaiki Taro Inoue, arXiv:0903.1541; P. Hunt and S. Sarkar, arXiv:0807.4508; S. Alexander, T. Biswas, A. Notari and D. Vaid, arXiv:071220370; J. Garcia-Bellido J and T. Haugboelle, 2008; T. Clifton, P. G. Ferreira, K. Land K, 2008.]. The fits need a large void ($\sim 0.75 - 1.5$ Gpc) and $h \sim 0.5$. This disagrees with the HST measurement $h \sim 0.71$. However, recollect that the local value of H can be smaller at large distance in an inhomogeneous LTB model e.g. at the surface of last scattering due to $t_{sls} = t_{\Sigma}(r) > t_{sls}(\text{LFRW})$.
- The void model must fit the acoustical power spectrum, the matter SDSS power spectrum and the BAO bump data.

An important data fitting is for the peculiar velocities of galaxy clusters obtained from kinematic Sunyaev-Zeldovich measurements [D. Garfinkle, 2009]. Moreover, it is possible for the LTB void model to explain the large discrepancy observed in smaller z kSZ measurements which disagree with the Λ CDM model by factors of 5 up to 300 Mpc.

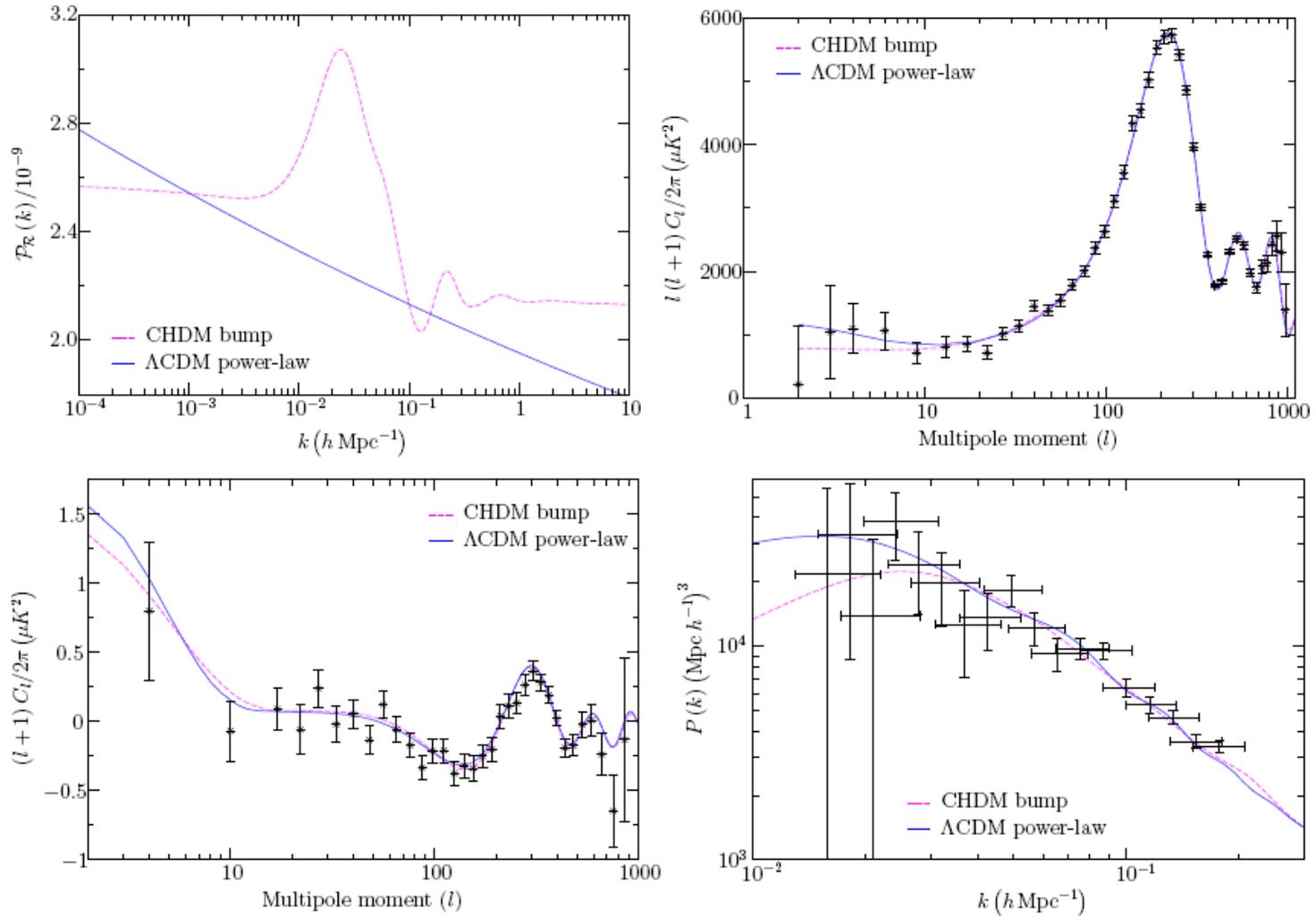


Figure 1. The top left panel shows the primordial perturbation spectrum for the CHDM bump model (with $n_1 = 12$ and $n_2 = 13$) and for the Λ CDM power-law model with $n_s \simeq 0.96$. The top right and bottom left panels show the best-fits for both models to the WMAP-5 TT and TE spectra, while the bottom right panel shows the best-fits to the SDSS galaxy power spectrum. **10% hot neutrinos used with $m\nu \sim 0.5 \text{ eV}$.**

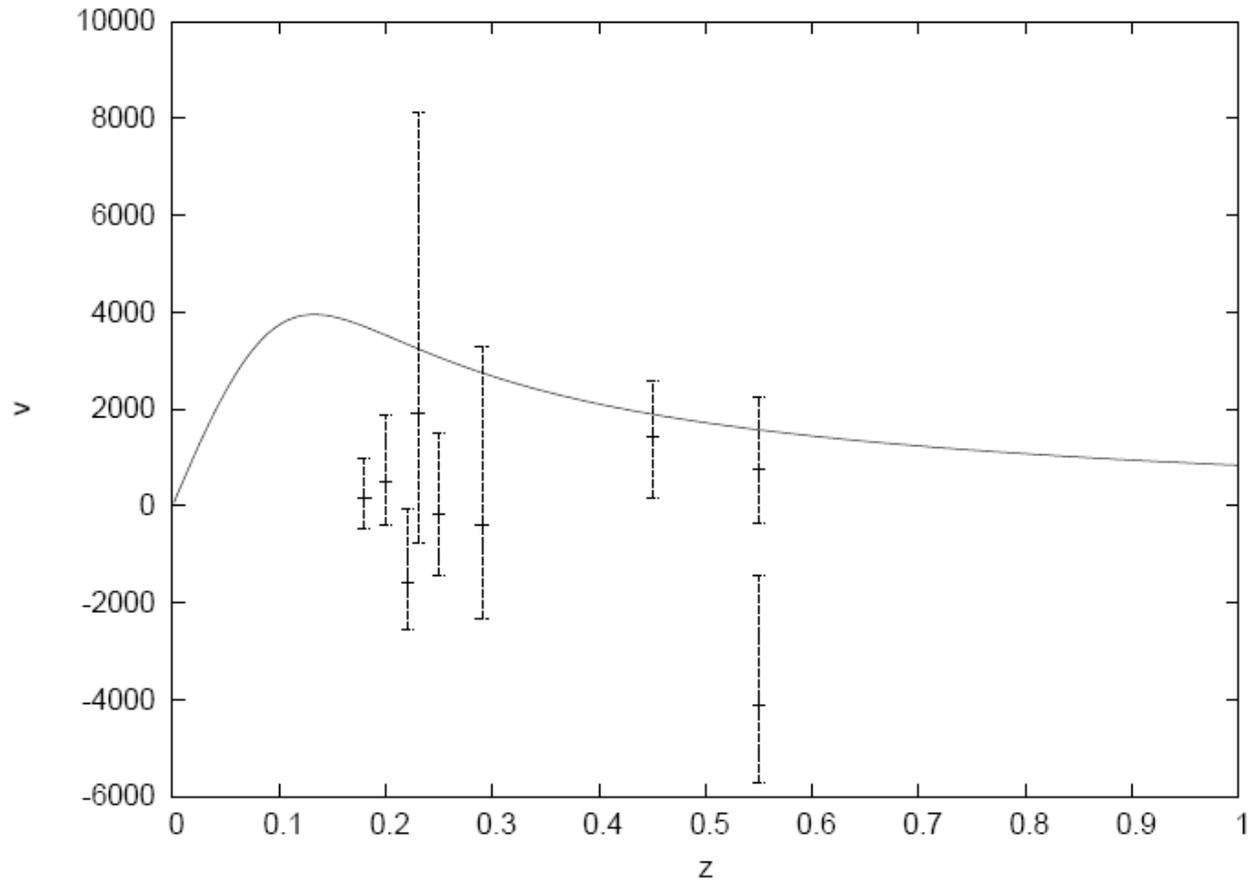
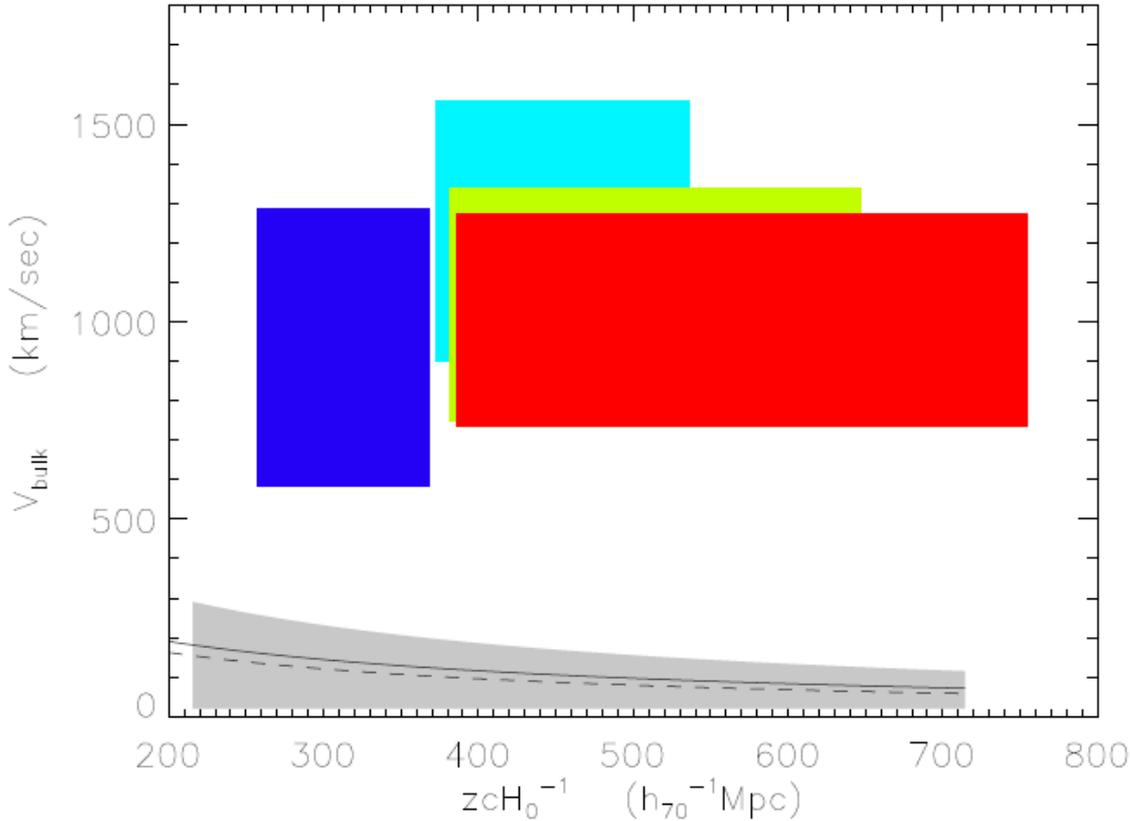


Figure 1. Plot of velocity (in units of km/s) *vs* redshift for the $\Omega_M = 0.3$ LTB model

D. Garfinkle, 2009. Data from B. Benson et al. 2003.



Blue corresponds to $z < 0.12$, cyan to $z < 0.16$, green to $z < 0.2$ and red to $z < 0.25$. Thick solid/dashed lines correspond to the rms bulk velocity for the concordance CDM model for top-hat/Gaussian windows. Black shaded regions shows the 95% confidence level of the model. A. Kashlinsky et al. 2009; Richard Watkins et al. 2008. Late time ISW may be better fitted with void model.

6. Alternative Exact Inhomogeneous Solutions

- There exist other exact inhomogeneous solutions of Einstein's field equations.
- Spherically symmetric Stephane solutions with density ρ and pressure p .
- Szekeres solutions. In general, these exact solutions have no special symmetry.
- Szafron solution with density ρ and pressure p . In general, these exact solutions have no special symmetry and will not violate the Copernican Principle.
- In general, the Szafron solution has no special symmetry, but a sub-solution has cylindrical symmetry. The latter solution leads to a possible non-uniform inflation period and can still potentially fit the supernova data [JWM, [arXiv:astro-ph/0606124](https://arxiv.org/abs/0606124)].

7. Conclusions

Do we really need **Dark Energy** ??

NO

Lemaitre-Tolman-Bondi

Exact Solution for Void

dust fluid and spherical symmetry

Violates Copernican Principle in space

Can the Void be large enough?

Can fit the WMAP data with $h \sim 0.5$ and $\Lambda = 0$. No Dark Energy!

YES

Violates Copernican principle in time t

→ need dark energy

Fine-tuning of cosmological constant Λ . Cannot detect Dark Energy!

Potential problem with cluster velocity... kSZ

Does fit CMB WMAP data with $h \sim 0.71$